ACKNOWLEDGMENT

The authors are grateful to the Fossil Energy Division, Department of Energy for financial support.

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Manuscript received August 11, 1977; revision received November 8, and accepted December 1, 1977.

Fully Developed Laminar Incompressible Flow in an Eccentric Annulus

J. TERRY WILSON

Department of Mathematics
San Jacinto College, Pasadena, Texas

Past investigators of laminar incompressible fluid flow in eccentric annular ducts have concentrated on the fully developed problem. Each investigation yielded a different expression for the fully developed velocity, which was then used to determine other information pertinent to fully developed flow. Snyder and Goldstein (1965), for example, used their expression to determine local wall shear stress and various friction factors. A summary of heat transfer and fluid flow research in eccentric annular and various other ducts has been compiled by Shah and London (1971). The purpose of this paper is to present a different representation of the fully developed velocity which proves quite useful in the analysis of the entrance region problem. A brief description of the general entrance region problem and an explanation of the role of our representation in solving this problem for an eccentric annular duct follow.

In 1964, Sparrow et al. proposed a linearized version of the entrance region problem for a straight duct with an arbitrary cross section. A method of solution, which represented the entrance region velocity as the sum of the fully developed velocity and a difference velocity, was also proposed and, in fact, applied to the parallel plate and circular tube problems. The application of their method to other geometries is warranted by the close agreement of their analytical results with experimental data.

The author has recently solved the equation which governs the difference velocity in an eccentric annular duct using the Galerkin method. The coordinate functions used to represent the difference velocity are used in this paper to represent the fully developed velocity. Thus, the entrance region velocity can be expressed in terms of a single set of coordinate functions. Furthermore, our representation allows for the calculation of all Galerkin inner products, difference velocity coefficients, and fully developed velocity coefficients in closed form.

0001-1541-78-9995-0733-$00.75.

ANALYSIS

The geometry under consideration is shown in Figure 1. The equation governing the flow is given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{dp}{dz} \tag{1}$$

where the pressure gradient dp/dz and the viscosity μ are constants. We impose the nonslip boundary condition $u = 0$ on C, the duct walls. Symmetry considerations allow us to solve Equation (1 only in, say, the lower half of the annulus. It also follows from symmetry that $\partial u/\partial y = 0$ on B, the plane of symmetry of the duct. If we define

$$c_1 = \frac{r_2^2 - r_1^2 - e^2}{2e}$$

and

$$c_2 = \frac{r_2^2 - r_1^2 + e^2}{2e}$$

then the bipolar transformation given by

$$\xi = \frac{1}{2}\ln\left[\frac{(x+h)^2 + y^2}{(x-h)^2 + y^2}\right] - k_2$$

$$\eta = \arctan\left[\frac{-2hy}{x^2 + y^2 - h^2}\right]$$

maps the lower half of the annulus onto the rectangle shown in Figure 2. In terms of the $\xi - \eta$ coordinates, Equation (1) is given by

$$\frac{\partial^2 v}{\partial \xi^2} + \frac{\partial^2 v}{\partial \eta^2} = \frac{-1}{[\cosh(\xi + k_2) - \cos\eta]^2} \tag{2}$$

where

$$v = \frac{-u}{\dfrac{h^2}{\mu}\dfrac{dp}{dz}}$$

is the dimensionless velocity. The boundary conditions for

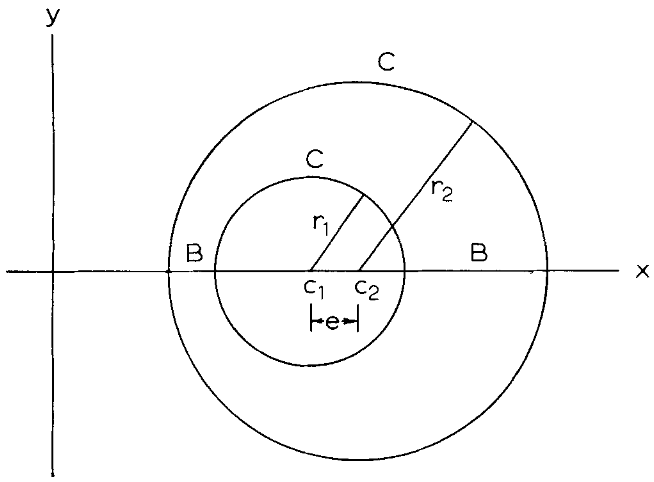

Fig. 1. Eccentric annulus geometry.

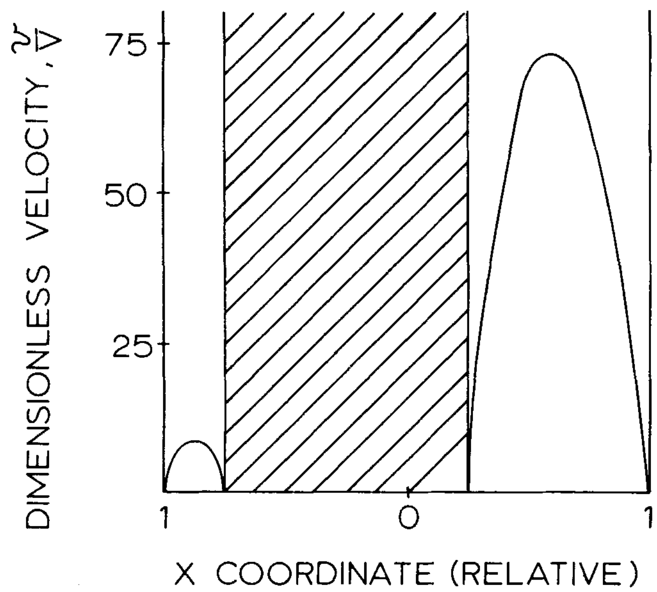

Fig. 3. Dimensionless velocity profile.

v are $v = 0$ on $\overline{C}$ and $\partial v/\partial \eta = 0$ on $\overline{B}$. Since the sequence

$$\left\{ \sqrt{\frac{2}{\pi k_3}} \sin \frac{n\pi\xi}{k_3} \right\} \bigcup_{n=1}^{\infty} \left\{ \frac{2}{\sqrt{\pi k_3}} \sin \frac{n\pi\xi}{k_3} \cos m\eta \right\}_{m,n=1}^{\infty}$$

is a complete orthonormal sequence for the rectangle, the function $v(\xi, \eta)$ can be represented in the form

$$v(\xi,\eta) = \sum_{n=1}^{\infty} a_n \sin \frac{n\pi\xi}{k_3} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} b_{mn} \sin\frac{n\pi\xi}{k_3} \cos m\eta \tag{3}$$

The velocity $v(\xi, \eta)$ satisfies the boundary conditions, since each of the coordinate functions does. Substitution of (3) into Equation (2) yields, using the orthogonality of the coordinate functions

$$a_n = \frac{2k_3}{n^2\pi_3} \int_0^{\pi} \int_0^{k_3} \frac{\sin \frac{n\pi\xi}{k_3}}{[\cosh(\xi + k_2) - \cos\eta]^2} d\xi \, d\eta \tag{4a}$$

$$b_{mn} = \frac{4k_3}{\pi[(n\pi)^2 + (mk_3)^2]} \int_0^{\pi} \int_0^{k_3} \frac{\sin \frac{n\pi\xi}{k_3} \cos m\eta}{[\cosh(\xi + k_2) - \cos\eta]^2} d\xi \, d\eta \tag{4b}$$

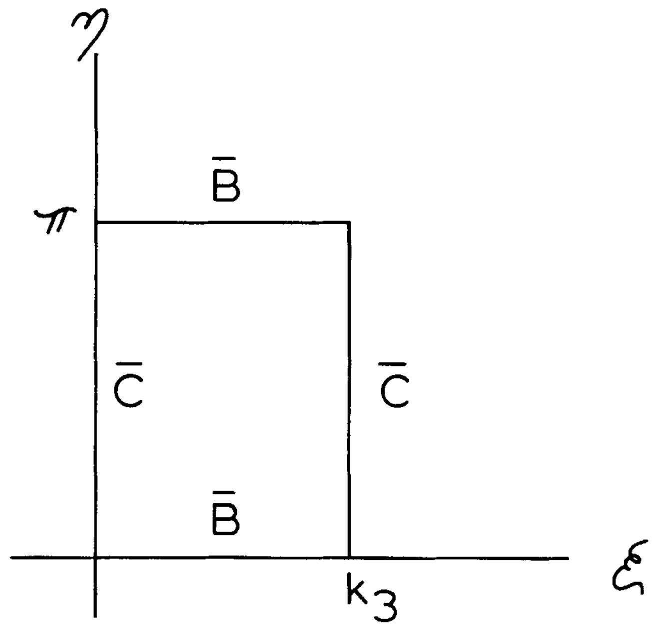

Fig. 2. Transformed eccentric annulus geometry.

If we use the residue theorem from complex variables, the integrals (4a) and (4b) can be evaluated in closed form. After a series of tedious calculations, we obtain

$$a_n = \frac{-1}{n\pi} \left\{ [B_2 + (-1)^{n+1} B_1] + 2 \sum_{j=1}^{\infty} \left\{ \frac{(n\pi)^2}{(2jk_3)^2 + (n\pi)^2} [A_2^{2j} + (-1)^{n+1} A_1^{2j}] \right\} \right\} \tag{4a}$$

$$b_{mn} = \frac{-2n\pi}{(n\pi)^2 + (mk_3)^2} \left\{ [A_2^m B_2 + (-1)^{n+1} A_1^m B_1] + 2 \sum_{j=1}^{\infty} \left\{ \frac{(mk_3)^2 + (n\pi)^2}{(mk_3 + 2jk_3)^2 + (n\pi)^2} [A_2^{m+2j} + (-1)^{n+1} A_1^{m+2j}] \right\} \right\} \tag{4b}$$

RESULTS

The velocity profile along the plane (line) of symmetry B for the duct defined by $r_1 = 4$, $r_2 = 8$, and $e = 2$ is shown in Figure 3. The velocity plotted is the quotient of the fully developed velocity $v(x, 0)$ and the average fully developed velocity over an arbitrary cross section of the duct. A straightforward calculation shows that the average velocity V is given by

$$V = \frac{2}{\pi} \sum_{\text{odd } n} \frac{a_n}{n}$$

In order to verify the accuracy of our fully developed velocity representation, Snyder's representation was used to calculate the velocity values at the same points used to construct Figure 3. These two sets of velocity values were found to be in excellent agreement; in fact, the absolute value of the relative differences was less than 0.04%.

CONCLUSION

In addition to providing an accurate representation of the fully developed velocity, our representation has other significant features. The first and most important feature is that it proves extremely useful in the analysis of the entrance region problem. Secondly, with this representation it is possible to obtain a closed-form expression for the average fully developed velocity. Finally, all of the constants in this representation can be written in terms of the duct input parameters: r_1, r_2, and e. Thus, the effect of the input parameters on the fully developed velocity can be analyzed.

ACKNOWLEDGMENT

The author wishes to thank Ms. Susan Hedman for preparing the figures.

NOTATION

a_n, b_{mn} = Fourier coefficients defined by Equation (4)

$A_i = \frac{c_i}{r_i} - \sqrt{\left(\frac{c_i}{r_i}\right)^2 - 1} \quad (i = 1, 2)$

$B, \overline{B}$ = plane of symmetry of duct

$B_i = 1 - 1 \Big/ \sqrt{1 - \left(\frac{r_i}{c_i}\right)^2} \quad (i = 1, 2)$

c_1, c_2 = duct centers defined by Figure 1

$C, \overline{C}$ = duct walls

$e\ (= c_2 - c_1)$ = eccentricity

$h = \sqrt{c_i^2 - r_i^2} \quad (i = 1, 2)$

$k_i = \ln \left\{ \frac{\sqrt{c_i + r_i} + \sqrt{c_i - r_i}}{\sqrt{c_i + r_i} - \sqrt{c_i - r_i}} \right\} \quad (i = 1, 2);$

$k_3 = k_1 - k_2$

p = pressure

r_1, r_2 = duct radii

u = fully developed velocity (v = dimensionless fully developed velocity)

V = average (dimensionless) fully developed velocity

x, y, z = Cartesian coordinates

ξ, η = transformed coordinates for x and y, respectively

μ = viscosity

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Manuscript received February 7, 1977; revision received October 12, and accepted December 7, 1977.

Recycle Effects on the Distribution of ATP Regeneration Catalysts

C. Y. CHOI and D. D. PERLMUTTER

Department of Chemical and Biochemical Engineering
University of Pennsylvania
Philadelphia, Pennsylvania 19104

Although ATP is one of the more important cofactors in biochemistry (Kalckar, 1969; Mahler and Cordes, 1971), its high cost has prevented common use in biosynthetic reactions and has stimulated interest in regeneration from its lower phosphorylated counterparts such as ADP, AMP, and adenosine. Among the possible means for the regeneration of ATP, the extracellular bi-enzymatic processes were proposed as most economic (Langer et al., 1976, 1977; Yang et al., 1976; Pace et al., 1976), and these prospects became more attractive with the advent of immobilized enzyme systems (Zaborsky, 1973). One particular path suggested by Langer et al. (1976) uses the phosphotransferases, that is, adenylate kinase and acetate kinase, to catalyze the reactions

$$\text{AMP} + \text{MgATP} \underset{}{\overset{\text{Adenylate Kinase}}{\rightleftarrows}} \text{ADP} + \text{MgADP} \quad (1)$$

$$\text{AcP} + \text{MgADP} \xrightarrow{\text{Acetate Kinase}} \text{Ac} + \text{MgATP} \quad (2)$$

The immobilized forms of these two enzyme catalysts are suited to use in a packed-bed tubular reactor system where they may be distributed along the reactor to achieve the optimal yield for ATP. Such optimization problems have been studied by King et al. (1972) and Choi and Perlmutter (1977); however, the net forward rate of the adenylate kinase reaction will be very small even with the optimal catalyst distribution policy, because the adenylate kinase reaction is readily reversible and the initial feed to the system will usually contain a very small fraction of ATP left over from a prior biosynthetic reaction. Depending on the initial concentrations of the system chemical species, this reaction may even proceed in the reverse direction when less than optimal catalyst distributions are used. In view of such difficulties, it is of interest to examine the effect of recycle on the objective of obtaining maximum yields of ATP. The following reports such a study for a representative set of initial concentrations and system parameters.

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